Symmetry-dependent defect structures in soft-mode turbulence

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(Received 15 July 2009; published 2 October 2009)

In the soft-mode turbulence (SMT) in electroconvection of homeotropic nematic systems, which is a kind of spatiotemporal chaos induced by nonlinear interaction between two two-dimensional (2D) *XY* fields, the Nambu-Goldstone modes, and the convective modes, a curious line structure called *blackline* has been discovered. We measured the density of the blackline as a function of control parameters, ac voltage, and frequency. By detailed observations and analysis, it is clarified that the blackline is a structure of the nematic director in the *x*-*y* plane and includes a sequence of point defects. We discussed similarity with the density of the blackline and that of the point defect in the conventional 2D *XY* model. The occurrence of this type of defects is only due to the symmetry in the SMT and independent of the properties of fluctuations.

DOI: 10.1103/PhysRevE.80.041701 PACS number(s): 61.30.Jf, 47.52.+j, 61.30.Gd, 64.60.Cn

Defects have been ubiquitously researched in many physical systems [1,2]. The research of defects is very important since the properties of a system can be reflected by the presence of defects. They are usually related to symmetry of the system [3,4]. For example in spin models, symmetries in the two-dimensional (2D) *XY* and 2D Ising models generate point [5,6] and line defects [7], respectively. The appearance of defects in a phenomenon called defect turbulence observed in electroconvective systems [8] and in other spatially extended nonlinear systems [9–11] has been much investigated. Here, we are working on soft-mode turbulence (SMT) in electroconvection of a homeotropic nematic system [12]. In this paper, we report a curious structure called the *blackline* observed in the SMT (see Fig. 1). We find the blackline is a type of defects or disclinations. We are interested to investigate this structure since the SMT has the same dimension and the degree of freedom of vector field as the conventional 2D *XY* model [13,14]. The purpose of this paper is an investigation of new aspect of defect related to the symmetry in the SMT.

In a homeotropic system without external fields, the nematic director is perpendicular to the electrodes. When an ac voltage V beyond the so-called Fréedericksz transition threshold V_F is applied to the system, the director tilts with respect to the z axis. This state is called the Fréedericksz one. The projection of the tilted director on the x-y plane is called the $\mathbf{C}(\mathbf{r})$ director, where $\mathbf{r}=(x,y)$. Since the transition spontaneously breaks the continuous rotational symmetry on the x-y plane, the rotation of $\mathbf{C}(\mathbf{r})$ behaves as a Nambu-Goldstone mode [15]. Beyond the threshold for convection V_c, the resulting local convective mode $\mathbf{q}(\mathbf{r})$ which nonlinearly interacts with the Nambu-Goldstone mode leads to spatiotemporal chaos called SMT [12]. With respect to the ac frequency f of the applied voltage, there are two types of SMT pattern namely oblique rolls (ORs) and normal rolls (NRs) [16] in which the corresponding ac frequency is below and beyond the so-called Lifshitz frequency f_L, respectively [17,18].

The experimental setup and the sample cell using a nematic liquid crystal p-methoxy-benziliden-p′-n-buthyl-annyline (MBBA) were similar to ones in the previous paper [14]. The cell thickness was $d=53\pm2$ μm. The shape of electrodes was circle with the diameter 13.0 mm. The experimental temperature was stabilized to be 30.00 ± 0.05 °C. The dielectric constant $\epsilon_\parallel$ and the electric conductivity $\sigma_\parallel$ of the material in the cell were 4.7 ± 0.1 and $7.7\pm0.1\times10^{-7}$ Ω^{-1} m^{-1}, respectively. An alternating voltage $V_{ac}(t)=\sqrt{2}V\cos(2\pi ft)$ was applied perpendicular to the sample. We define a normalized control parameter $\varepsilon\equiv(V/V_c)^2-1$. The pattern images in the x-y plane were captured using a charge-coupled device camera (QImaging Retiga 2000R-Sy) mounted on a microscope and a software (QCapture Pro v. 5). A cross-polarized set (crossed-Nicol) can be installed in the microscope or removed easily. The size of the captured images was 1.14 mm × 1.14 mm(1000 pixel × 1000 pixel). The image analysis was performed by custom software.

Figure 2(a) shows a shadowgraph image of the SMT with a blackline for $\varepsilon=0.10$ and $f<f_L$. From the shadowgraph image, the convective wave vector $\mathbf{q}(\mathbf{r})$ can be observed [19,20]. It is guessed that the blackline is related to $\mathbf{C}(\mathbf{r})$

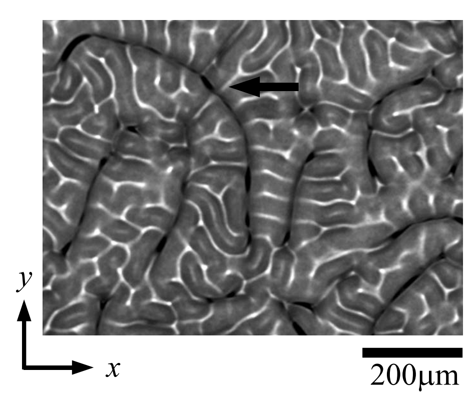

FIG. 1. Blacklines in the SMT. One of them is shown by a black arrow. The bright lines correspond to the convective patterns of the SMT.

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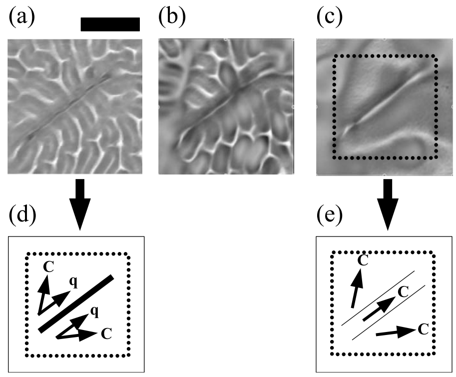

FIG. 2. (a) A shadowgraph image of SMT with a blackline for $\varepsilon=0.10$ and $f<f_{\mathrm{L}}$. The image is observed at about 10 min later after jumping of voltage from the Freedericksz state into $\varepsilon=0.1$. The scale bar corresponds to 100 μm. (b) Corresponding crossed-Nicol image for (a). (c) Crossed-Nicol image at $t\approx 5$ s after jumping the ac voltage into $\varepsilon<0$ corresponding to the Fréedericksz state. (d) Schematic image of the structure of blackline corresponding to (a). (e) Schematic image corresponding to (c).

director since the blackline cannot be recognized as a convective pattern. Generally, in order to show $\mathbf{C}(\mathbf{r})$ pattern, the crossed-Nicol was used. The intensity of $\mathbf{C}(\mathbf{r})$ pattern can be written as $I(\mathbf{r})\propto \sin^2[2\alpha(\mathbf{r})]$, where α is the azimuthal angle of $\mathbf{C}(\mathbf{r})$ director. However the convective and $\mathbf{C}(\mathbf{r})$ patterns near the blackline are mixed up because of the mix-up of the shadowgraph and the crossed-Nicol images [see Fig. 2(b)]. To analyze the blackline, the ac voltage was jumped into the Fréedericksz state below V_c by the following reason. There are two relaxation times for the jumping: One is τ_θ which is the relaxation time of the tilted angle of the nematic director with respect to the z axis and corresponds to convective amplitude. The other relaxation time is τ_α for azimuthal angle α of director $\mathbf{C}(\mathbf{r})=C(\cos[\alpha(\mathbf{r})],\sin[\alpha(\mathbf{r})])$ in the x-y plane. By considering that $\tau_\theta\approx O(1)$ s and $\tau_\alpha\approx O(10)$ s, we jumped the ac voltage below V_c and observed the change in the pattern after $t_1\approx 5s$, that is $\tau_\theta<t_1<\tau_\alpha$. After jumping, we found that the $\mathbf{C}(\mathbf{r})$ pattern becomes clear to be observed because the convective patterns disappear [see Fig. 2(c)]. From Fig. 2(c) we could draw the schematic image of $\mathbf{C}(\mathbf{r})$ around the blackline [see Fig. 2(e)]. Hence, from Fig. 2(e) the blackline can be regarded as a line defect of $\mathbf{C}(\mathbf{r})$ director. Finally, the schematic image of $\mathbf{C}(\mathbf{r})$ and $\mathbf{q}(\mathbf{r})$ for Fig. 2(a) is shown in Fig. 2(d). The schematic image shows that the angle between $\mathbf{C}(\mathbf{r})$ director and the wave vector $\mathbf{q}(\mathbf{r})$ of is nonzero, which corresponds to the OR.

For more detailed investigation of this line defect, the above experiment was also performed for longer time. We traced the evolution of a blackline and found that the blackline changes into one or more point defects observed in the Fréedericksz state. For the reverse case, that is a jump from $V_c>V>V_{\mathrm{F}}$ into $V>V_c$, we also found that such point defects change into a blackline. Here, we make a schematic image for these evolutions. Figure 3(a) shows that in the Fréedericksz state, there are two neighboring point defects with different strength [21] (+1 and −1) as confirmed by the rotation of the crossed-Nicol. The arrows show the direction of $\mathbf{C}(\mathbf{r})$ director around the point defects. The dark and the

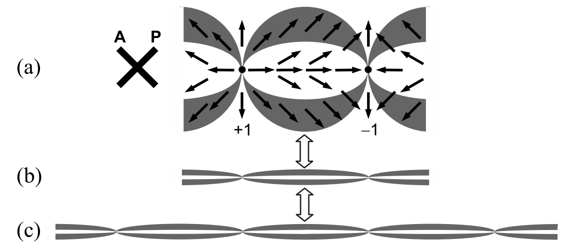

FIG. 3. Evolution from point defects into a blackline and vice versa. (a) Two point defects with different strength (+1 and −1) in the Fréedericksz state. The black arrows show the direction of the azimuthal angle α of $\mathbf{C}(\mathbf{r})$-director. (b) Blackline originated from the point defects by compressing of the dark region. (c) Longer blackline. See text for details.

bright domains represent the intensity $I(\mathbf{r})$ for the $\mathbf{C}(\mathbf{r})$ field under the crossed-Nicol. When the point defects changes into a blackline, the dark domains are compressed and finally a blackline appears [see Fig. 3(b)]. On the other hand, the dark domains expand when the blackline changes into point defects. Therefore the blackline includes point defects and then for more rigorous conclusion, it can be regarded as a pseudoline defect. Now we have a question why the blackline appears in the form of line. The reason will be described later.

We observed that nucleation and annihilation of point defects can occur in the blackline, as the length of blackline increases and the decreases, respectively [see Fig. 3(c)]. Therefore it is expected that there is a quantitative relation between blacklines and point defects. We measured the total length of blackline in an observation area. By a jumping of voltage below V_c similarly to the above-mentioned experiment, when the blacklines change into point defects, immediately we measured the total number of the point defects in the same observation area with the increase in ε. Here, we define densities of blackline ρ_{BL} and point defect ρ_{PD} as the total length of all blacklines and total number of point defects divided by the observation area, respectively. Figure 4 shows that independently of ε, ρ_{BL} is proportional to ρ_{PD}. Therefore the increase in the total length of blacklines accompanies with the increase in number of point defects.

From the above relation shown in Fig. 4, we are interested

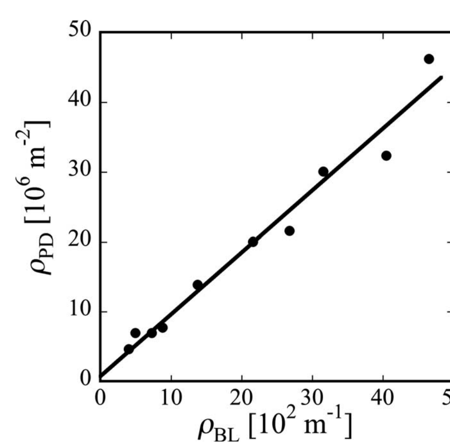

FIG. 4. Density of blackline ρ_{BL} versus density of point defect ρ_{PD}. The range of ε is from 0.05 to 0.4.

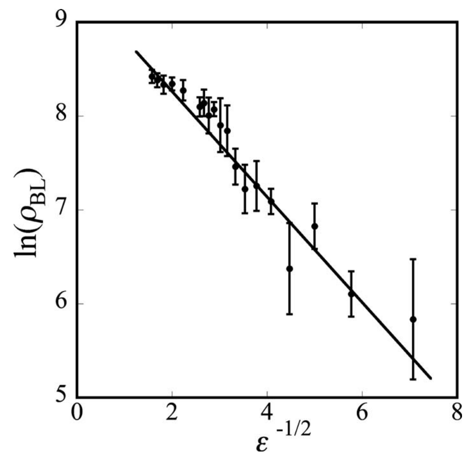

FIG. 5. Linear relation between $\ln(\rho_{BL})$ and $\varepsilon^{-1/2}$. The ac frequency is 100 Hz.

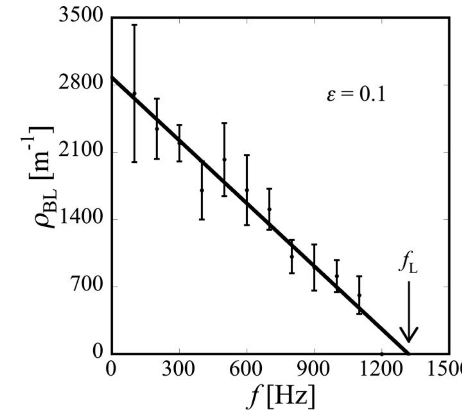

FIG. 6. Blackline density ρ_{BL} as a function of ac frequency f.

to compare between ρ_{BL} in the SMT and density of point defect ρ_{XY} in the conventional 2D *XY* model. The reason for the comparison is that both systems have the same dimension and degree of freedom of vector fields. In the higher temperature regime of the conventional 2D *XY* model, the density of point defect $\rho_{XY} \propto \exp(-b\eta^{-1/2})$, where $\eta = T/T_{KT} - 1 \geq 0$, b is a positive constant, T is temperature, and T_{KT} is the Kosterlitz-Thouless transition temperature [6]. For the blackline, we measured ε dependence of ρ_{BL}. We found that ρ_{BL} can be fitted into $\exp(-c\varepsilon^{-1/2})$ [a linear relation between $\ln(\rho_{BL})$ and $\varepsilon^{-1/2}$, see Fig. 5], where c is a positive constant. This means that the relation between ρ_{BL} and ε is similar to the relation between ρ_{XY} and η. By using the analogy, the density of blackline is in a good agreement with that of point defect in the conventional 2D *XY* model. The good agreement surprises us because the sources of the fluctuations are different. Namely, the SMT is generated by the nonthermal fluctuations, whereas in the conventional 2D *XY* the thermal ones play an important role. Thus the symmetry of the system and the degree of freedom are more important than the properties of fluctuations in this phenomenon.

We also found another important property of the blackline by measuring the density ρ_{BL} as a function of another control parameter, namely, ac frequency. It has been reported that ac frequency f becomes a control parameter for the SMT pattern [13]. We found that ρ_{BL} linearly decreases for the increase in f, as shown in Fig. 6. The density is zero at the Lifshitz frequency f_L [13]. This means that the blackline is found only in the OR regime. The existence of the blackline is related to symmetry breaking explained as follows. As mentioned above, the blackline is a kind of structure of $\mathbf{C}(\mathbf{r})$ director. For the NR and OR regimes, the wave vector $\mathbf{q}(\mathbf{r})$ is parallel and oblique with $\mathbf{C}(\mathbf{r})$, respectively. Therefore, $\mathbf{C}(\mathbf{r}) \times \mathbf{q}(\mathbf{r})$ changes from zero (the NR regime) to nonzero (the OR regime) at the transition point. This means that the reflection symmetry of the interaction between the $\mathbf{C}(\mathbf{r})$ director and $\mathbf{q}(\mathbf{r})$ is broken at the Lifshitz frequency.

The above result can more be understood by considering $\|\mathbf{C}(\mathbf{r}) \times \mathbf{q}(\mathbf{r})\| \propto \sin\theta(\mathbf{r})$, where $\theta(\mathbf{r})$ is the angle between $\mathbf{C}(\mathbf{r})$ and $\mathbf{q}(\mathbf{r})$. For the OR regime, on the other hand, since $\mathbf{C}(\mathbf{r})$ is not parallel to $\mathbf{q}(\mathbf{r})$, then in both sides across a blackline $\mathbf{C}(\mathbf{r}) \times \mathbf{q}(\mathbf{r})$ can take different nonzero values, positive $(\sin|\theta|)$ and negative $(-\sin|\theta|)$ [see Fig. 2(d)]. Therefore, from the viewpoint of the relation between the two 2D *XY* fields [$\mathbf{C}(\mathbf{r})$ director and $\mathbf{q}(\mathbf{r})$], the SMT in the OR regime can be recognized as a 2D Ising-like system. This analogy can answer why the blackline which consists of point defects appears in the form of line in the 2D *XY* system.

Finally we would like to conclude the present results as follows. In a 2D *XY* system called the soft-mode turbulence observed in a homeotropic nematic system, a line structure called blackline appears. The blackline is a structure of $\mathbf{C}(\mathbf{r})$ director. We found that the blackline actually includes point defects and therefore it can be regarded as a pseudoline defect. Then we reveal that the statistical property of the blackline is in a good agreement with that of point defect in the conventional 2D *XY* model. Finally the existence of the blackline is related to the symmetry breaking in the relation between Nambu-Goldstone modes and the convective modes.

We thank M. I. Tribelsky for fruitful discussion and K. Takeuchi for useful comment. This work is partially supported by Grant-in-Aid for Scientific Research (Grants No. 17340119, No. 20111003, No. 21340110, and No. 21540391) from the Ministry of Education, Culture, Sport, Science, and Technology of Japan and the Japan Society for the Promotion of Science (JSPS). R.A. acknowledges the support by a Grant-in-Aid for Scientific Research (Grant No. 20.08333) from JSPS.

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